

IT/CS-4507**B. E. (Fourth Semester) EXAMINATION, June, 2003**

(Information Technology & Computer Science Engg.)

DISCRETE STRUCTURES

(IT/CS-4507)

Time : Three Hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : Attempt any *five* questions. All questions carry equal marks.

1. (a) Let R be a binary relation on the set of all integers such that $R = \{(a, b) \mid (a - b) \text{ is an even integer}\}$. Is R an equivalence relation? 5
- (b) Explain the principle of inclusion and exclusion. 4
- (c) By principle of mathematical induction prove that :

$$2 \cdot 7^n + 3 \cdot 5^n - 5$$
is divisible by 24 for all natural numbers n . 7
2. (a) Let $A = \{a, b, c, d\}$ and $P(A)$ its power set. Draw Hasse diagram of $(P(A), \subseteq)$. 7
- (b) Explain any *three* of the following : 3 each
 - (i) Partitions
 - (ii) Chains and Antichains
 - (iii) Pigeon hole principle
 - (iv) Lattice

3. (a) Using truth table, determine whether each of the following is a tautology, a contingency or an absurdity? 5 each
 - (i) $(p \rightarrow q) \wedge (p \vee q)$
 - (ii) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
- (b) State whether the argument given below is valid or not valid. If it is valid, identify the Tautology or Tautologies used : 6

I will become famous or I will be a writer

I will not be a writer

 $\therefore$ I will become famous

4. (a) For the finite state machine shown below, find all equivalent states and obtain an equivalent finite state machine with the smallest number of states : 10

State	INPUT		Output
	0	1	
$\Rightarrow$ A	F	B	0
B	D	C	0
C	G	B	0
D	E	A	1
E	D	A	0
F	A	G	1
G	C	H	1
H	A	H	1

- (b) Show that the language :

$$L = \{a^k \mid k = i^2, i \geq 1\}$$

is not a finite state language. 6

5. (a) Explain any *two* of the following : 6
 - (i) Eulerian Path and Circuit
 - (ii) Rooted tree
 - (iii) Binary Search Tree

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- (b) Write an algorithm for shortest path in weighted graph and use it to find shortest path for a to z in the graph shown in fig. 1 where numbers associated with the edges are the weights. 4, 6

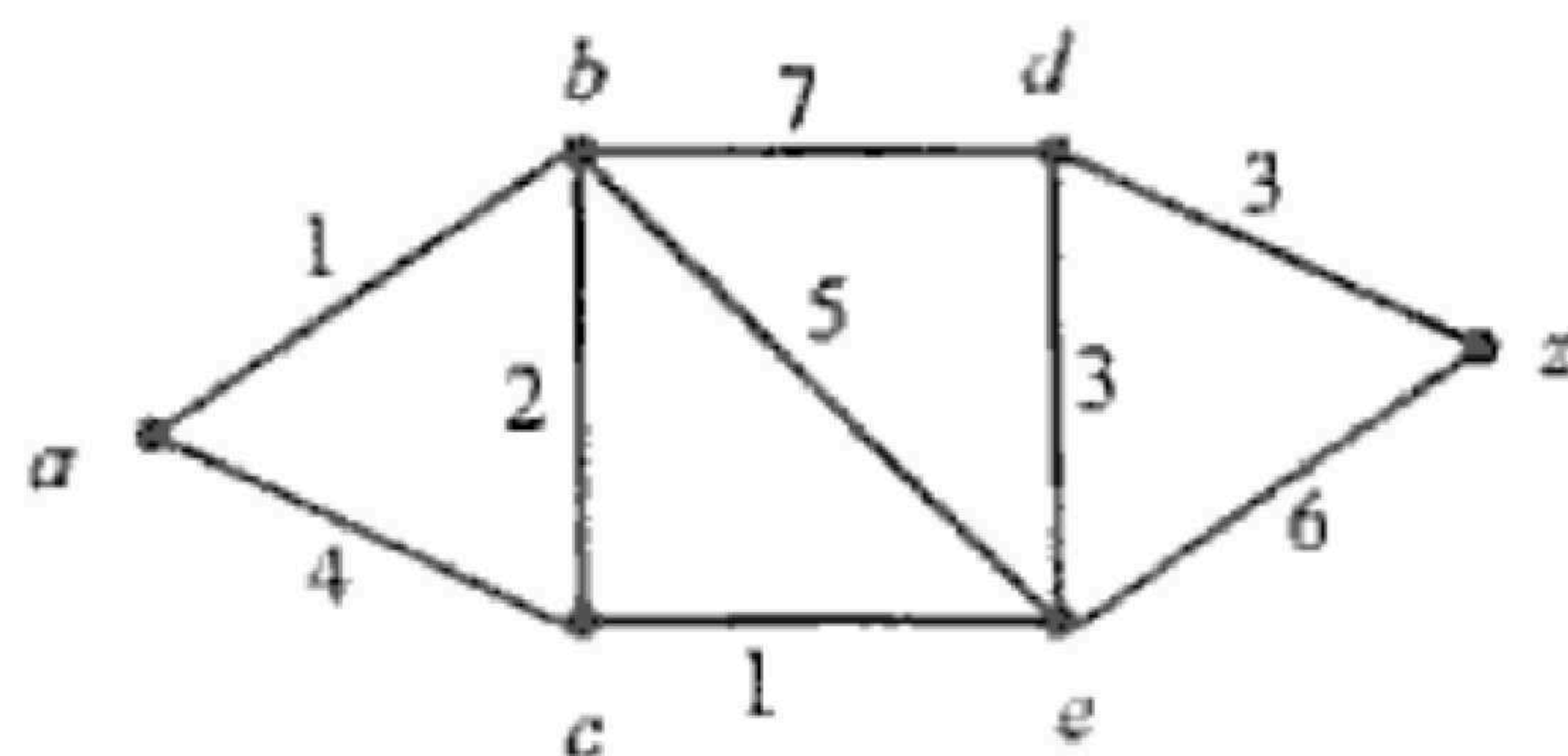


Fig. 1

6. (a) Write an algorithm to find minimum spanning tree and use it to find a minimum spanning tree for the graph shown in fig. 2. 4, 6

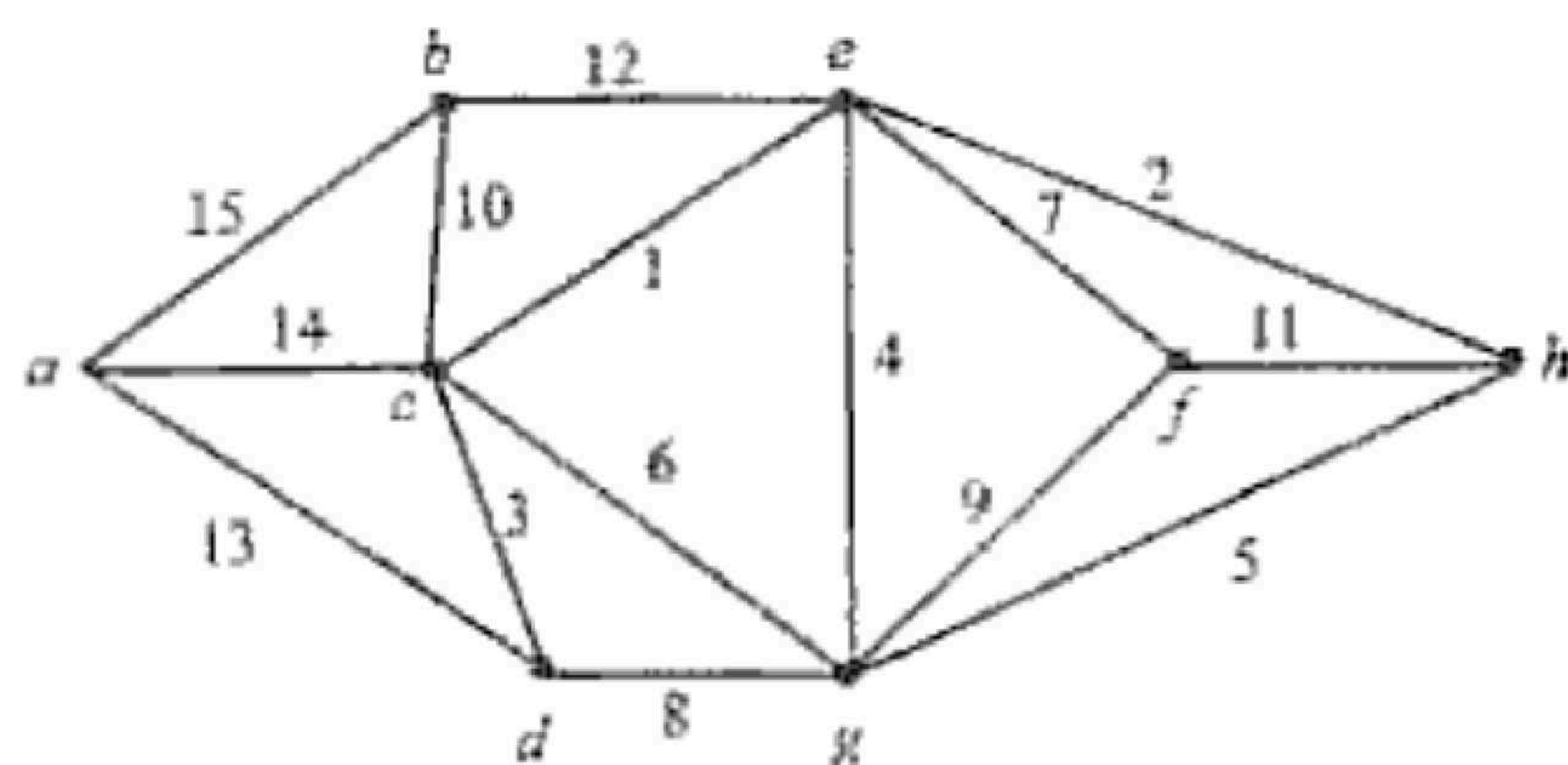


Fig. 2

- (b) Let a , b and c be numeric functions such that $a * b = c$. Given that : 6

$$a_r = \begin{cases} 1, & r = 0 \\ 2, & r = 1 \\ 0, & r \geq 2 \end{cases}$$

$$c_r = \begin{cases} 1, & r = 0 \\ 0, & r \geq 1 \end{cases}$$

find b .

6

P. T. O.

7. (a) Solve the recurrence relation : 8

$$a_r - 4a_{r-1} + 4a_{r-2} = (r+1)^2, r \geq 2$$

- (b) Write short notes on any two of the following : 4 each

- (i) Isomorphism and Automorphism
- (ii) Cosets
- (iii) Groups and semigroups

8. (a) Let G be the set of the non-zero real numbers and let $a * b = \frac{ab}{2}$ then show that $(G, *)$ is an abelian group. 8

- (b) If R is a ring, then for all $a, b, c \in R$ show that : 4 each

(i) $ao = oa = 0$

(ii) $a(-b) = -(ab) = (-a)b$