

CS/IT-405

B. E. (Fourth Semester) EXAMINATION, Dec., 2003

(Common for CS & IT Engg.)

DISCRETE STRUCTURES

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 35

Note : Answer any five questions. All questions carry equal marks.

1. (i) Prove that if A and B are finite sets :

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

- (ii) Define Mathematical Induction. Use it to prove :

$$2 + 4 + 6 + \dots + 2n = n(n + 1).$$

- (iii) Given $A = \{1, 2\}$, $B = \{x, y, z\}$ and $C = \{3, 4\}$.

Find $A \times B \times C$.

2. (i) Consider the relation R on the set Z of integers :

$$x \equiv y \pmod{m}, m \text{ is an integer } > 1.$$

Show that it defines an equivalence relation on Z.

- (ii) Consider the set Z of integers. Define $a R b$ by $b = a^r$ for some positive integer r. Show that R is a partial order on Z.

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3. (i) Obtain the converse, inverse and contrapositive of the conditional statement $p \rightarrow q$.
- (ii) Write the negation of the statement :
If she works, she will earn money.
- (iii) Test the validity of the argument :
If I study, then I will not fail in Mathematics.
If I do not play basket ball, then I will study.
But I failed in Mathematics
Therefore, I must have played basket ball.
- (iv) Show that $p \vee \sim(p \wedge r)$ is equivalent to $(p \vee \sim q) \vee \sim r$.
- (v) Ajoy loves Nina.
Ajoy loves Anita too.
Does Ajoy really loves Nina ?
4. (i) Let $A = \{a, b\}$. Construct an automation M which will accept the language $L(M) = \{a^r b^s : r > 0, s > 0\}$.
- (ii) Let M be the finite state machine with state table :

F	a	b
S ₀	S ₁ , x	S ₂ , y
S ₁	S ₃ , y	S ₁ , z
S ₂	S ₁ , z	S ₀ , x
S ₃	S ₀ , z	S ₂ , x

- (a) Find the input set A, the state set S the output set Z and the initial state.
- (b) Draw the transitional diagram of M.
- (c) Let $w = aababababab$ is an input word. Find the output word V.

5. (i) Prove that a finite connected graph G is Eulerian if and only if each vertex has even degree.
 (ii) Define planar graph, regular and bipartite graph. Draw the graph $K_{2,5}$.
 (iii) Define a tree, spanning tree, minimum spanning tree.
6. (i) Let a, b, c be numeric functions such that $a * b = c$.
 Given :

$$a_n = \begin{cases} 1 & r = 0 \\ 2 & r = 1, \\ 0 & r \geq 2 \end{cases} \quad C_r = \begin{cases} 1 & r = 0 \\ 0 & r \geq 1 \end{cases}$$

Determine b .

- (ii) Solve the recurrence relation :
 $a_r - 7a_{r-1} + 10a_{r-2} = 0$, given $a_0 = 0, a_1 = b$.
7. (i) Let H be a normal subgroup of group G . Then the cosets of H in G form a group under coset multiplication :

$$(aH)(bH) = abH.$$

Prove it.

- (ii) Prove that a finite integral domain is a field.
8. Write short notes on any *three* of the following :
- (i) A relational model for data bases
 - (ii) A pigeon hole principle
 - (iii) Shortest path in weighted graph
 - (iv) Lattice, distributive lattice, complement of an element