Total No. of Questions : 8] *[Total No. of Printed Pages : 2*

Roll No .....

CS/IT-224

B.E., III Semester

Examination, December 2016

Choice Based Credit System (CBCS)

Discrete Structure

Time : Three Hours

Maximum Marks : 60

Note: i) Attempt any five questions.

ii) All questions carry equal marks.

1. a) Let A, B, C be any three sets, then prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

   b) Show that if R_1 and R_2 are equivalence relations on A, then $R_1 \cap R_2$ is an equivalence relation on A.

2. a) Let $f: R \to R$ be defined by $f(x) = 3x + 4$, show that f is one-one and onto. Give a formula that defines f^{-1}.

   b) Prove by the method of mathematical induction that $7^{2n} + 2^{3n-3} \cdot 3^{n-1}$ is divisible by 25 for all $n \in N$.

3. a) Show that the algebraic structure $\left(\left\{a + b\sqrt{2} : a, b \in I\right\}, +\right)$ forms a group.

   b) Prove that every field is an integral domain.

4. a) Obtain disjunctive normal form of $P \vee (\sim P \to (q \vee (q \to \sim r)))$

   b) Show that $((p \vee q) \wedge \sim(\sim p \wedge (\sim q \vee \sim r))) \vee (\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)$ is a tautology.

5. a) Find a deterministic Finite-State Machine that recognizes the set:

$$L = \left\{(01)^i 1^{2j} \mid i \geq 1, j \geq 1\right\}$$

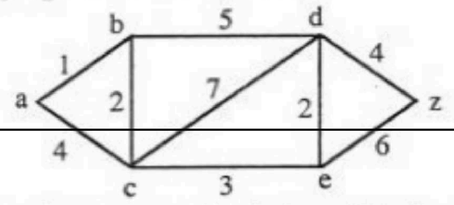
b) For the finite state machine shown below, find all equivalent states and obtain an equivalent finite state machine with the smallest number of states:

State	Input		Output
	0	1	
A	F	B	0
B	D	C	0
C	G	B	0
D	E	A	1
E	D	A	0
F	A	G	1
G	C	H	1
H	A	H	1

6. a) Write a short note on:
   i) Isomorphism of groups
   ii) Universal and existential quantifiers

   b) Explain:
   i) Hamiltonian paths and circuits
   ii) Graph coloring

7. a) Determine shortest path between vertices 'a' and 'z' in the graph shown below:



   b) Prove that every chain is a distributive lattice.

8. a) Find total solution for the recurrence relation

$$a_r - 4a_{r-1} + 4a_{r-2} = (r+1)2^r$$

   Given $a_0 = 1, a_1 = 2, a_2 = 3$

   b) Describe :
   i) Hasse diagram ii) Binomial theorem

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