

**CS/EI/IT-405** EI

**B. E. (Fourth Semester) EXAMINATION, Dec., 2004**

(Common for CS, EI & IT Engg.)

**DISCRETE STRUCTURE**

*Time : Three Hours*

*Maximum Marks : 100*

*Minimum Pass Marks : 35*

**Note :** Attempt any five questions. All questions carry equal marks.

1. (a) If R and S are equivalence relation in a set X, then prove that  $R \cap S$  is also an equivalence relation in the set X.

(b) Show that :

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}; n \geq 1$$

by Mathematical induction.

2. (a) Show that the relation defined by " $\leq$ " on the set of all natural numbers is reflexive and transitive but not symmetric.

(b) Show that the following propositions are tautologies :

(i)  $(p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

(ii)  $\{(p \vee \sim q) \wedge (\sim p \vee \sim p)\} \vee q$

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3. (a) For each of the sets described below, find a deterministic finite state machine that recognizes the set :

$$L = \{(01)^i 1^{2j} \mid i \geq 1, j \geq 1\}$$

(b) For the finite state machine shown in the figure :

| State | Input |   | Output |
|-------|-------|---|--------|
|       | 0     | 1 |        |
| A     | B     | F | 0      |
| B     | B     | C | 1      |
| C     | D     | C | 1      |
| D     | A     | E | 1      |
| E     | A     | D | 0      |
| F     | F     | D | 0      |

(i) List all 0-equivalent states.

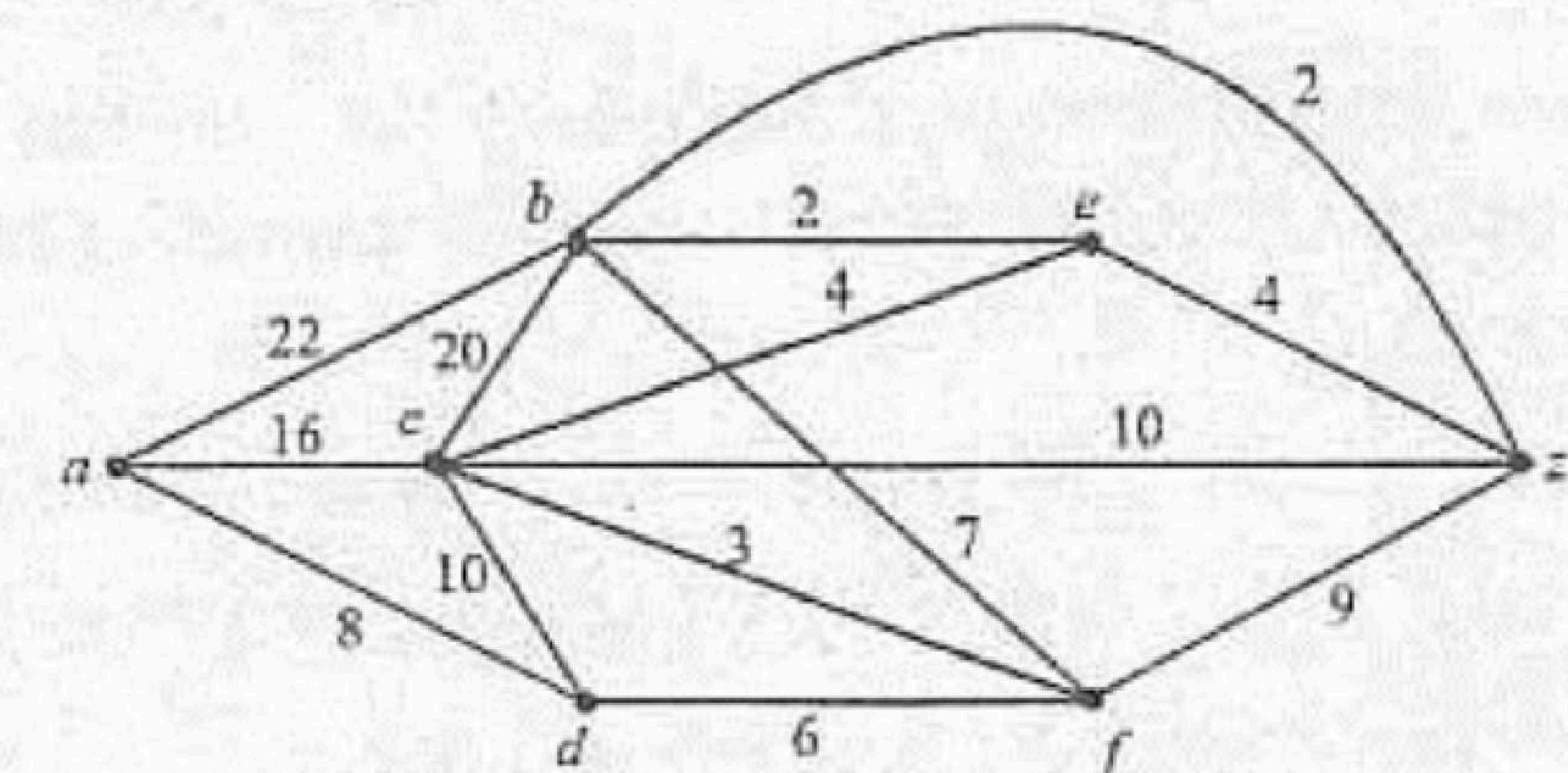
(ii) Find all equivalent states and obtain an equivalent finite state machine with the smallest number of states.

4. (a) Prove that the following statement is logically equivalent :

$$(p \Rightarrow q) \vee r \equiv (p \vee r) \Rightarrow (q \vee r)$$

(b) Define walk, path and circuit in a directed graph. What is the metric in a graph ?

5. (a) Determine shortest path between 'a' and 'z' in the graph shown below, where the numbers associated with the edges are the distance between vertices :





(b) A graph is given by the following adjacency matrix :

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 2 \\ 2 & 3 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}$$

Check whether it is connected or not.

6. (a) Solve the recurrence relation :

$$a_r + 6a_{r-1} + 9a_{r-2} = 3$$

given that  $a_0 = 0, a_1 = 1$ .

(b) Determine the generating function of the numeric function  $a_r$  where

$$a_r = \begin{cases} 2^r, & \text{if } r \text{ is even} \\ -2^r, & \text{if } r \text{ is odd} \end{cases}$$

7. (a) Find total solution for the difference equation :

$$a_r - 4a_{r-1} + 4a_{r-2} = (r+1)2^r$$

Given  $a_0 = 1, a_1 = 2, a_2 = 3$

(b) Write generating function for the following discrete numeric functions :

(i) 2, 3, 5, 9, 17, 33, ....

(ii) 2, 5, 13, 35, ....

8. (a) Let  $(A, *)$  be a semigroup. Further more for every  $a$  and  $b$  in  $A$ , if  $a \neq b$  then :

$$a * b \neq b * a;$$

Show that for every  $a, b$  in  $A$ ,  $a * b * a = a$ .

(b) Show that kernel of homomorphism of ring  $R$  to  $R'$  is an ideal.